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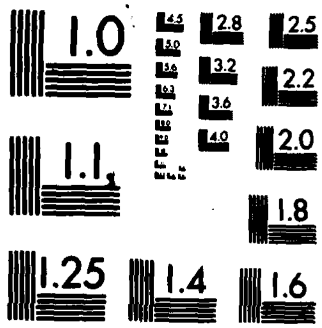
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ROBOT MANIPULATOR CONTROL

J.F. BLACKBURN

7 March 1983

UNITED STATES OF AMERICA

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This report presents a synthetic approach for calculating the control of robot manipulators. The initial control problem is broken down into linear control and modelling problems. The approach allows derivation of numerous schemes (adaptive or not) of control proposed in the literature and suggests new schemes. It is shown that the problem of modelling is difficult but is less crucial if one can synthesize robust controls that are not sensitive to errors of modelling.

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ROBOT MANIPULATOR CONTROL

Claude Samson and J.M. Ibarra Zannatha, Institut de Recherche en Informatique et Systemes Aleatoires, Rennes, France, have developed a general approach to the problem of controlling robot manipulators. Samson and Zannatha have separated the larger problem into two issues: controlling a known time-invariant linear system and modelling.

The two sub-problems are related when control robustness is considered with respect to modelling errors. Samson and Zannatha's approach allows derivation of several adaptive or nonadaptive control schemes described in the literature and suggests new schemes. It applies to continuous and discrete cases and shows possibilities for computing controls from a set of operational coordinates of the manipulator.

This report is a translated and abbreviated version of a paper titled "Sur La Commande Des Manipulator" by Samson and Zannatha.

Mathematical Model of a Manipulator With N Degrees of Freedom

We begin with the general form of a mathematical model of a manipulator:

$$M(x)\,\ddot{x} + V(x,\dot{x}) = u \tag{2.1}$$

in which x is the vector of n generalized coordinates, $\dot{x}$ is its time derivative (speed vector), $\ddot{x}$ is its second derivative (acceleration vector), and M(x) is the kinetic energy or inertia matrix, which is always positive definite. $V(x,\dot{x})$ is the vector combining the Coriolis and centrifugal forces; u is the force and moments vector acting on the manipulator, other than $V(x,\dot{x})$. One can decompose u into the sum of three terms:

$$u = u_g(x) + u_f(\dot{x}) + u_a \tag{2.2}$$

where $u_g(x)$ is the force of gravity, $u_f(\dot{x})$ is friction, and u_a represents forces produced by the motors on the movement of the robot. The force of interest here is u_a.

If we combine (2.1) and (2.2), the manipulator model can be written:

$$M(x)\,\ddot{x}+N(x,\dot{x}) = u_a \tag{2.3}$$

where

$$N(x,\dot{x}) = V(x,\dot{x}) - u_g(x) - u_f(\dot{x}). \tag{2.4}$$

Dividing the Initial Control Problem Into Linear Control and Modelling Problems

Equation 2.3 is replaced by the following system of equations equivalent to it.

$$\ddot{x} = u' \quad (2.5a)$$

$$u_{a1} = M(x)u' \quad (2.5b)$$

$$u_{a2} = N(x,\dot{x}) \quad (2.5c)$$

$$u_a = u_{a1} + u_{a2}. \quad (2.5d)$$

If $M(x)$ and $N(x,\dot{x})$ are known or easily calculated at each instant, then it is sufficient to calculate the control u' associated with an invariant linear system of second order (2.5a), and then to calculate u_a using (2.5b, c, and d).

If $M(x)$ and $N(x,\dot{x})$ are not known and are too complex to calculate at each instant, we can work with approximate matrices $\hat{M}$ and $\hat{N}$ and use the following working model derived from the mathematical model (2.5):

$$\ddot{x} = u' \quad (2.6a)$$

$$u_{a1} = \hat{M}\ u' \quad (2.6b)$$

$$u_{a2} = \hat{N} \quad (2.6c)$$

$$u_a = u_{a1} + u_{a2}. \quad (2.6d)$$

Thus the problem of control of the manipulator is the sum of two sub-problems:

1. A linear control problem associated with equation (2.6a), in which we can use the classic theory of linear and invariant systems control to compute u'.

2. A modelling problem associated with computing the matrices $\hat{M}$ and $\hat{N}$.

The control will be only as accurate as the approximation of the matrices $\hat{M}$ and $\hat{N}$ to the matrices $M(x)$ and $N(\dot{x},x)$.

Although the two problems require different techniques, they are not totally independent. Errors associated with the modelling problem may have more or less serious consequences depending on the type of control chosen for u'.

Two Strategies of Control for Following a Reference Trajectory: Consigned Control and Control With Reference Model

The strategy for control u_a is determined by the computation of u'. To simplify the study we recall the principle of two methods most often used for the control of manipulators: consigned control and control with reference model.

Consigned Control

When the manipulator is first at rest $[\dot{x}(t_o) = o]$, we wish to have x

return to a consigned x_c according to a certain dynamic. We consider the following control u':

$$u' = -K_p(x-x_c) - K_v\dot{x} \tag{4.1}$$

(K_p and K_v are diagonal matrices).

We obtain, following (2.6a), the transfer function:

$$x(s) = (K_p/(s^2+K_v s+K_p))x_c(s). \tag{4.2}$$

The time constant and the overshoot of the indexed response of the system (4.2) are regulated by the choice of gains K_p and K_v. The gains are therefore entirely determined by the dynamic of the return to the consigned value.

Control With Reference Model

The method consists of giving the equation of a reference system (or reference model) of the same dimensions as the model of the manipulator, and computing the control u' so as to cancel the error ε between the position vector x of the manipulator and the position vector x_r of the reference model.

The choice of the reference model must be compatible with the structure of the manipulator. There should be a bounded control u' allowing cancellation of the error ε (see H. Erzberger, "Analysis and Design of Model Following Control Systems by State-Space Techniques" Proceedings of the Joint Automatic Control Conference [1968], pp 572-581). Referring to equation (2.6a), an evident choice for the reference model is:

$$\ddot{x}_r = u_r. \tag{4.3}$$

Let

$$\varepsilon = x - x_r \tag{4.4}$$

subtracting (4.3) from (2.6a) we obtain

$$\ddot{\varepsilon} = u' - u_r. \tag{4.5}$$

A possible choice for the control u' is then:

$$u' = u_r - L_p\varepsilon - L_v\dot{\varepsilon} \tag{4.6}$$

where L_p and L_v are diagonal matrices. The dynamic equation of the error ε is given by:

$$\begin{pmatrix} \dot{\varepsilon} \\ \ddot{\varepsilon} \end{pmatrix} = \left(\begin{array}{c|c} O & I_n \\ \hline -L_p & -L_v \end{array}\right) \begin{pmatrix} \varepsilon \\ \dot{\varepsilon} \end{pmatrix} . \tag{4.7}$$

Using the Routh-Hurwitz criteria, it can be shown that (4.7) is the equation of an exponentially stable system, and the elements of the diagonal matrices L_p and L_v are positive. The choice of higher gains leads to faster exponential convergence toward zero.

The control in the reference model acts uniquely as a regulator because its role is to bring the error to zero. The reference control u_r is required to follow a reference trajectory. Unlike the consigned control method, the choice of gains L_p and L_v are subject to no constraints coming from the dynamic of the reference trajectory. It can be shown that the method of consigned control is a special case of the reference model method obtained by choosing

$$u_r = - K_p x_r - K_v \dot{x}_r + K_p x_c$$

with

$$L_p = K_p$$
$$L_v = K_v.$$

<u>Control Schemes as a Function of Modelling</u>

The relations (2.6b, c, and d) and (4.1) can be combined to obtain the consigned control:

$$u_a = -\hat{M}K_p(x-x_c) - \hat{M}K_v\dot{x} + \hat{N}. \tag{5.1}$$

The relation (2.6b, c, and d) with (4.6) give the reference model control:

$$u_a = - \hat{M}L_p\varepsilon - \hat{M}L_v\dot{\varepsilon} + \hat{M}u_r + \hat{N}. \tag{5.2}$$

First Modelling

The classic controls with constant coefficients are those computed from simplified models of manipulators using the following hypothesis:

H_1 : $M(x) = \hat{M}$: constant diagonal matrix

$$\left.\begin{array}{l} H_2 : u_g(x) \approx 0;\ u_f(\dot{x}) \approx 0 \\ H_3 : V(x,\dot{x}) \approx 0 \end{array}\right\} \longrightarrow \hat{N} = 0$$

Hypothesis H_1 is justified for small displacements and when the N degrees of freedom of the manipulator are practically decoupled. It also supposes that the matrix of inertia is only slightly sensitive to the variations of mass carried. Hypothesis H_2 indicates that the gravity terms (often compensated in part by counterweights), and the friction terms are negligible. Finally, hypothesis H_3 is justified if the displacement speed $\dot{x}$ of the manipulator is small.

Associating this model with (5.2) gives a nonclassical scheme of model reference control with constant coefficients.

Second Modelling

Using the mathematical model of the manipulator we calculate for each point of the trajectory the matrices $M(x)$ and $N(x,\dot{x})$, which gives:

$$\hat{M} = M(x)$$
$$\hat{N} = N(x,\dot{x}).$$

Controls using the model are called dynamic controls; it is possible to have dynamic control with a reference model. The computation of the matrices $M(x)$ and $N(x,\dot{x})$ is complex and in some situations cannot be done--for example, when the mass and shape of the object carried vary greatly and are unknown.

Third Modelling

We identify on-line the matrices M and N beginning with a parameterization of the matrices established in advance. The controls thus synthesized are adaptive. More generally, we can design by adaptive control all controls requiring measurements made on the manipulator to identify its structure or the control law (see, for example, S. Dubowsky and D.J. Des Forges, "The Application of Model-Referenced Adaptive Control to Robotic Manipulators," <u>Journal of Dynamic Systems, Measurement and Control</u>, Vol 101 [September 1979], pp 193-200). According to the definition, the dynamic controls previously discussed are not adaptive. The mathematical model of the manipulator from which they are computed will be a function of the measurements of x and $\dot{x}$.

The idea of adaptive control is in general motivated by two considerations:

1. No mathematical model of the manipulator completely reflects the reality because of the difficulty of modelling terms (such as friction) and changes in the course of use (for example, picking up a load).

2. An exact model of the manipulator is less important for control than an accurate identification of the structure of the manipulator arising from its principal characteristics.

There is no general method for on-line identification of M and N. There are, however, techniques for estimating recursively a vector θ of constant

parameters, or slowly changing variables when we have a relation of the following form (see L. Ljung, and T. Soderstrom, Theory and Practice of Recursive Identification [Cambridge, MA: MIT Press, 1982]):

$$y(t) = \phi(t)^T \theta, \tag{5.3}$$

where $y(t)$ is a known vector, and $\phi(t)^T$ is a known matrix of observation with dimensions compatible with those of $y(t)$ and θ.

Such a relation will normally be obtained after parameterization of the dynamic equation of the manipulator:

$$M \ddot{x} + N = u_a, \tag{5.4}$$

which leads to the adaptive control schemes called "indirect." We now give several examples of parameterization.

Example 1. The vector θ is made up of the totality of components of the matrices M and N. Relation (5.4) can be written in the form (5.3) with $y(t) = u_a(t)$ and $\phi(t)^T$ made up of unity and the components of acceleration $\ddot{x}(t)$. In the continuous case, the parameterization assumes knowledge of the acceleration $\ddot{x}(t)$. In using the symmetry of the matrix M, the number of components entering into θ can be reduced substantially. Also, the knowledge of zero components of M, from consideration of the geometry of the manipulator, reduces the size of θ.

With such parameterization, the matrices $\hat{M}(t)$ and $\hat{N}(t)$ are directly obtained from the identified vector $\hat{\theta}(t)$.

Example 2. If the gravity and friction forces are negligible, use of (2.4) reduces $N(x,\dot{x})$ to $V(x,\dot{x})$, and we can write the following (see R. Horowitz and M. Tomizuka, "An Adaptive Control Scheme for Mechanical Manipulators Compensation of Nonlinearity and Decoupling Control," American Society of Mechanical Engineers (ASME), Paper No. 80, Wa/DSC-6 [1980]):

$$V(x,\dot{x}) = \begin{pmatrix} \dot{x}^T V^1 (x) \dot{x} \\ \\ \dot{x}^T V^n (x) \dot{x} \end{pmatrix} \tag{5.5}$$

where the matrices $V^i(x)$ are symmetric and only depend on the position x.

A possible parameterization is then obtained by putting into the vector θ the unknown components of M and of the matrices $V^i(x)$. The parameterization leads to a vector θ, generally larger than in the preceding example. Its advantage is that the components entering in θ are not functions of the speed $\dot{x}$, and for movements of small amplitude we can expect the vector $\theta(x)$ to be nearly constant.

Example 3. For a given load, the parameters in the mathematical model of the manipulator are fixed and we can try to identify them on-line and enter them into the vector θ. For a given configuration, θ is constant and thus independent of x and $\dot{x}$ and the computed control.

Identifying θ in the above case involves computation at each instant of the coefficients multiplying the components of θ in the mathematical model of the manipulator. The computed controls are dynamic adaptive controls, which in some cases are too complex for easy computation.

Observation. The set of adaptive controls that we have considered are studied in more detail in J.M. Ibarra Zannatha, "Sur La Commanc Des Robtos Manipulateurs" (Doctoral thesis, Univ. of Rennes, April 1982). The following observations are based on simulations during which many identification algorithms were tested.

1. The parameterizations in examples 1 and 2 do not permit easy identification of M and N (poorly or noisily followed). The poor identification is due to two main causes: the parameters that we seek to identify vary too rapidly with rapid movement of the manipulator, and the parameters depend on the intermediate values of x and $\dot{x}$. With the adaptive control depending on estimated parameters, the identification problem for the two parameterizations is ill conditioned.

2. In spite of the mediocre performance of the identification, the adaptive controls with the model of reference retain good performance when the gains L_p and L_v are large (the reference trajectory is closely followed). The interpretation of the phenomenon (observable in many existing studies) is important to understanding the problems in the control of manipulators. We attribute this largely to the robust character of the large gain control.

Robustness of Large Gain Control

A rapid heuristic study justifies the robust character of large gain controls with respect to errors in the modelling of the manipulator.

If we assume $\hat{M} = I$ (identity matrix) and $\hat{N} = 0$, the corresponding constant coefficient control with the model of reference is written according to (5.2):

$$u_a = -L_p\varepsilon - L_v\dot{\varepsilon} + u_r. \qquad (6.1)$$

Using the equations of the mathematical model of the manipulator and the reference model (4.3), we obtain the following equation for the error ε.

$$\ddot{\varepsilon} = M^{-1}\, u_a - u_r - M^{-1}N, \qquad (6.2)$$

and using (6.1) and (6.2)

$$\ddot{\varepsilon} = -M^{-1}L_p\varepsilon - M^{-1}L_v\dot{\varepsilon} + (M^{-1}-I)u_r - M^{-1}N. \qquad (6.3)$$

The choice of large gains L_p and L_v makes the term $-M^{-1}L_p\varepsilon - M^{-1}L_v\dot{\varepsilon}$

dominant over the term $(M^{-1}-I)u_r - M^{-1}N$ as soon as the position error ε or the velocity error $\dot{\varepsilon}$ becomes large. Then the equation (6.3) approximately reduces to:

$$\ddot{\varepsilon} = -M^{-1}L_p\varepsilon - M^{-1}L_v\dot{\varepsilon}. \qquad (6.4)$$

Suppose for simplification that M is constant or that its relative variation is small compared with that of ε (an hypothesis that we can verify later). M being positive definite, M^{-1} is also positive definite and consequently there exists an orthogonal matrix P such that:

$$P^TM^{-1}P = M_D^{-1} \quad : M_D \text{ is a diagonal matrix} \qquad (6.5a)$$

$$P^TP = I. \qquad (6.5b)$$

If we let $\varepsilon = P\xi$ (6.6)

and make use of (6.4) and 6.5b) we obtain:

$$\ddot{\xi} = -P^TM^{-1}L_pP\xi - P^TM^{-1}L_vP\dot{\xi} \qquad (6.7)$$

where L_p and L_v are chosen so that:

$$L_p = l_pI, \quad L_v = l_vI \quad (l_p>0, l_v>0). \qquad (6.8)$$

L_p *and* L_v *can permute with the matrix* P *in the* equation (6.7), and using (6.5a) we obtain:

$$\ddot{\xi} = -L'_p\xi - L'_v\dot{\xi} \qquad (6.9)$$

with $L'_p = l_pM_D^{-1}$, $L'_v = l_vM_D^{-1}$: positive diagonal matrices.

The relation (6.9) is a system of n decoupled equations of second order, exponentially stable. Consequently ξ converges to 0 exponentially in the same way as ε, following (6.6), the rate of convergence being the more rapid the larger the gains l_p and l_v.

The *preceding heuristic reasoning* although *not a* valid demonstration, helps us justify that the error of approximation ε cannot become large. If an error appears at any instant, then it diminishes exponentially and more rapidly as the gains l_p and l_v became larger.

We now examine the static error caused by the gravity term in N. Suppose that x and x_r converge toward constant values. Then $\dot{x} = 0$, $\dot{x}_r = 0$, $u_r = 0$ [in view of (4.3)], $N(x,\dot{x})$ is reduced to $-u_g(x)$ [in view of (2.4)] if we suppose the friction term is of the form $u_g(\dot{x}) = -k_f\dot{x})$, and equation (6.3) becomes

$$0 = - M^{-1} L_p \varepsilon_{stat} + M^{-1} u_g(x), \tag{6.4}$$

from which

$$\varepsilon_{stat} = \frac{u_g(x)}{l_p}, \tag{6.5}$$

showing that the static error due to the gravity term $u_g(x)$ is inversely proportional to the gain l_p.

Numerous simulations given by Ibarra-Zannatha confirm the robustness of large gain controls. The robustness property will be studied further.

Discretization of the Control Problem

Using relation (2.1) the equation of the mathematical model of the manipulator can also be written in the form of the equation of state:

$$\dot{X} = AX + B\ (x) u_a + C(x,\dot{x}) \tag{7.1}$$

with

$$X = \begin{pmatrix} x \\ \dot{x} \end{pmatrix}$$

$$A = \left(\begin{array}{c|c} 0 & I_n \\ \hline 0 & 0 \end{array}\right), B(x) = \begin{pmatrix} 0 \\ M^{-1}(x) \end{pmatrix}, C(x,\dot{x}) = \begin{pmatrix} 0 \\ -M^{-1}(x) N(x,\dot{x}) \end{pmatrix}$$

Let us assume u_a is numerically computed and maintained constant during a sampling step of length h. In integrating (7.1) over a sampling step we obtain:

$$X(t+h) = A_d X(t) + B_d u_a(t) + C_d \tag{7.2}$$

with

$$A_d = e^{Ah} = \left(\begin{array}{c|c} I_n & hI_n \\ \hline 0 & I_n \end{array}\right);$$

$$B_d = \int_t^{t+h} e^{A(t+h-s)}\ B(x(s))ds\ ;\ \text{and}$$

$$C_d = \int_t^{t+h} e^{A(t+h-s)}\ C[x(s),\ \dot{x}(s)]\ ds.$$

In general, it is not possible to calculate exactly B_d and C_d. However, if the sampling step h is chosen sufficiently small, we can assume that x and $\dot{x}$ vary little over this sampling step, as do B(x) and $C(x,\dot{x})$. B_d and C_d can then be approximated as follows:

$$B_d \cong \left(\int_t^{t+h} e^{A(t+h-s)}\, ds\right) B[x(t)] = \begin{pmatrix} \frac{h^2}{2} I \\ h\, I \end{pmatrix} M^{-1}[x(t)] \text{ and} \tag{7.3a}$$

$$C_d \cong \left(\int_t^{t+h} e^{A(t+h-s)}\, ds\right) C[x(t),\dot{x}(t)] =$$

$$- \begin{pmatrix} \frac{h^2 I}{2} \\ h\, I \end{pmatrix} M^{-1}[x(t)]N[x(t),\dot{x}(t)] \; . \tag{7.3b}$$

From equations (7.2) and (7.3), we find that the discretization of the problem of control of a manipulator leads to replacing the system of continuous equations (2.5) by the discrete system:

$$X(t+h) = A_d X(t) + \begin{pmatrix} \frac{h^2 I}{2} \\ h\, I \end{pmatrix} u'(t) \tag{7.4a}$$

$$u_{a1}(t) = M\ [x(t)]u'(t) \tag{7.4b}$$

$$u_{a2}(t) = N\ [x(t)\ ,\ \dot{x}(t)] \tag{7.4c}$$

$$u_a(t) = u_{a1}(t) + u_{a2}(t) \tag{7.4d}$$

The similarity between this system and the continuous system (2.5) shows that the approach we have considered in the continuous case also applies in the discrete case. The system shows in effect that the problem of computing the control u_a is decomposed into:

1. A control problem of a linear decoupled and discretized system (7.4a).
2. A modelling problem connected with the relations (7.4b) and (7.4c).

In particular, the property of robustness of control with large gains is preserved. We note also that the previous discretization eliminates the problem of determining the second derivative of x.

<u>Computation of the Control in the Frame of Reference of the Operational Coordinates</u>

In many applications, the user is interested chiefly in the control of the terminal arm of the manipulator. The position and speed of the arm are usually characterized by coordinates different from the manipulator's generalized coordinates, which are less practical to use. General coordinates

include, for example, the three coordinates in a fixed frame of reference of the extremity of the terminal arm, with the three Euler angles in this frame of reference characterizing the inclination of the terminal arm. To solve the control problem we have two possibilities:

1. Recompute the reference trajectory in the frame of reference of generalized coordinates, then compute the control u_a as before. The biggest difficulty with this method is in passing from the operational coordinates to the generalized coordinates, the inverse operation being generally much easier.

2. Compute directly the control u_a from the operational coordinates. The possibility returns to the recomputation of the model of the manipulator in the frame of reference of operational coordinates.

We show that for the second possibility, the approach developed earlier in this report remains applicable.

Let y be the vector of operational coordinates corresponding to the vector x of the generalized coordinates of the manipulator, and let f(.) be the function assumed with continuous derivative everywhere, which allows passing from x to y.

$$y = f(x) \tag{8.1}$$

Let J(x) be the Jacobian matrix of f defined as

$$J(x) = \frac{\partial f}{\partial x}. \tag{8.2}$$

Following (8.1) and (8.2)

$$\dot{y} = J(x)\dot{x}. \tag{8.3}$$

In taking the derivative of (8.3) we obtain:

$$\ddot{y} = J(x)\ \ddot{x} + W\ (x,\dot{x}) \tag{8.4}$$

with

$$W(x,\dot{x}) = \begin{pmatrix} \dot{x}^T W^1(x)\ \dot{x} \\ i \\ \dot{x}^T W^n(x)\ \dot{x} \end{pmatrix}, \tag{8.5}$$

where W^i is the derivative of the i^{th} line of J with respect to x. We suppose for simplification that the matrix J(x) has an inverse. (If J has no inverse we can use a pseudo-inverse of the matrix.)

Following (8.4) we have

$$\ddot{x} = J^{-1}(x)\ \ddot{y} - J^{-1}(x)\ W(x,\dot{x}). \tag{8.5}$$

Recalling equation (2.3) of the mathematical model of the manipulator

$$M(x)\ddot{x} + N(x,\dot{x}) = u_a. \tag{8.6}$$

Combining (8.5) and (8.6) we obtain

$$M'(x)\ddot{y} + N'(x,\dot{x}) = V_a \tag{8.7a}$$

$$u_a = J^T(x)\, V_a \tag{8.7b}$$

with

$$M'(x) = J^{-T}(x)\, M(x)\, J^{-1}(x) \text{ and} \tag{8.7c}$$

$$N'(x,\dot{x}) = -M'(x)W(x,\dot{x}) + J^{-T}(x)\, N(x,\dot{x}). \tag{8.7d}$$

Comparison of (8.7a) and (8.6) shows that the two relations have the same form. For instance, the *matrix of inertia* $M(x)$ is replaced by the matrix $M'(x)$, which is also positive definite if the Jacobian $J(x)$ has an inverse. It follows that the techniques of computing the control u_a (shown previously) are transposable to the computation of the control V_a using the operational coordinates of the manipulator. The control u_a that interests us is then computed from V_a according to relation (8.7b). The idea is found in M. Takegaki and S. Arimoto, "An Adaptive Trajectory Control of Manipulators," International Journal of Control, Vol 34, No. 2 (1981), pp 219-230.

Conclusion

This report has presented a synthetic approach for calculating the control of manipulators. The initial control problem is broken down into linear control and modelling problems. The approach allows derivation of numerous schemes (adaptive or not) of control proposed in the literature and suggests new schemes. It has been shown that the problem of modelling is difficult but is less crucial if one can synthesize robust controls that are not sensitive to errors of modelling. A heuristic study, which will later be refined, shows that large gain controls are robust and that controls with a reference model are naturally conditioned to function with large gains.

To give the study its correct weight, one must take a larger view and consider the control problem in a more general context than we have so far. Some difficulties do not appear with our mathematical model of the manipulator. The model is only a simplistic (although very useful) representation of the physical system of a manipulator. The model does not take into account, for example, the flexibility of the manipulator arms and the transmission organs. We have not considered possible errors in measuring x and $\dot{x}$, which could distort in an important way the performance of the control. It is easily conceivable that the set of the nonmodelled terms, which are fundamentally

different from those we have included, could change the basic structure of the system (in the case of the flexibilities when they are important), and make risky the control (in the case of noisy measurements). Such terms are not modelled now in studies of adaptive control for manipulators, but they can make the adaptation of control gains desirable and justify the study of more elaborate controls.

The above is, in our view, an important outcome of the present study, which must be viewed as a simple element of analysis, among others, of the general problem of the control of manipulators.


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